modulo p. In counting the number of incongruent fractions in the set (4) we must therefore consider the number of representations (5). We shall regard two representations

$$mn' + m'n = p, \quad m_1n_1' + m_1'n_1 = p$$

as the same if and only if $m = m_1$, $n' = n_1'$, $m' = m_1'$, $n = n_1$. If N is the number of representations of this type, then the relations (6) show that

$$N = K - (p - 1).$$

Now K by definition is equal to twice the number of distinct positive irreducible fractions whose numerators and denominators are each not greater than $\sqrt{p}$. Hence*

$$K = 4(\varphi(2) + \varphi(3) + \cdots + \varphi([\sqrt{p}])) + 2,$$

where $\varphi(k)$ denotes the number of integers $< k$ and prime to it. We therefore have

THEOREM III. *If p is a prime, then the number of representations of p in the form*

$$xy + x'y',$$

where x, y, x', y' are all positive integers $< \sqrt{p}$, is equal to

$$-(1 + p) + 4 \sum_{k=1}^{[\sqrt{p}]} \varphi(k).$$

PROOF OF A GENERAL THEOREM ON THE LINEAR DEPENDENCE OF p ANALYTIC FUNCTIONS OF A SINGLE VARIABLE.

BY MR. HAROLD MARSTON MORSE.

(Read before the American Mathematical Society, September 5, 1916.)

A PROOF of the following theorem has to my knowledge not been published to date. The theorem contains as a special case the ordinary theorem concerning the wronskian. Its usefulness in a general treatment of single-valued func-

* Lucas, Théorie des Nombres, p. 393.