Here $e_2 \oplus e_3 \neq e_3 \oplus e_2$.

$\overline{\text{III}}_b$.

$\oplus$	e_1	e_2	e_3
e_1	e_1	e_2	e_3
e_2	e_2	e_2	e_2
e_3	e_3	e_2	e_3

$\odot$	e_1	e_2	e_3
e_1	e_1	e_1	e_1
e_2	e_1	e_2	e_2
e_3	e_1	e_3	e_3

Here $e_2 \odot e_3 \neq e_3 \odot e_2$.

University of California,
March, 1916.

NOTE ON REGULAR TRANSFORMATIONS.

BY DR. L. L. SILVERMAN.

Let $u(x)$ be bounded and integrable, $0 \leqq x$, and $k(x, y)$ integrable in y for each x, $0 < y \leqq x$; then the transformation*

$$(1) \qquad v(x) = \alpha u(x) + \int_0^x k(x, s)u(s)ds$$

is regular if

$$\lim_{x=\infty} u(x)$$

implies the existence of

$$\lim_{x=\infty} v(x)$$

and the equality of the limits. The transformation (1), which depends on the number α and on the function $k(x, y)$, will be denoted by the symbol $[\alpha; k(x, y)]$. Examples of regular transformations are given by [1; 0], which is the identical transformation, and [0; $1/x$], which corresponds to the first Hölder mean. In a forthcoming paper† the author discusses conditions on α and $k(x, y)$ for the regularity of the transformation‡ (1), and proves the following theorem:||

Theorem 1. *A sufficient condition that* $k(x, y)$ *defined*, $0 < y \leqq x$, *and integrable in* y *for each* x, *correspond to a*

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* It is assumed that the improper integral converges; the lower limit of integration is taken zero for convenience.

† *Transactions*, vol. 17 (1916).

‡ The function $k(x, y)$ in (1) is $(1 - \alpha)$ times the function $k(x, y)$ in the article referred to.

|| See Theorem III in the article referred to; the numbers a and b of that theorem are here replaced by 0 and a respectively. The right-hand member of the last condition is $1 - \alpha$ instead of unity; see preceding footnote.