SINGULAR POINTS OF TRANSFORMATIONS AND TWO-PARAMETER FAMILIES OF CURVES.

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1. *Introduction.*

IN the *Transactions* for October, 1915, I discussed some singularities of a point transformation in three variables

$$(1) \qquad x = \phi(u, v, w), \quad y = \psi(u, v, w), \quad z = \chi(u, v, w).$$

Let a particular one of the singular points in question be denoted by P, and let S denote the surface through P in the uvw-space defined by setting the jacobian of the transformation equal to zero. The point P and the surface S are transformed by (1) into a point P_1 and surface S_1 in the xyz-space.

In the present paper there is found on the surface $S_1(x, y, z)$ a curve (d_1) which is the envelope of a one-parameter family of curves properly chosen from the two-parameter family (1). We find in the uvw-space that plane of directions which transforms into the direction of the curve (d_1) in the xyz-space.

2. *Initial Assumptions.*

Let us consider a real point transformation of three-space

$$(1) \qquad x = \phi(u, v, w), \quad y = \psi(u, v, w), \quad z = \chi(u, v, w)$$

with determinant

$$J(u, v, w) \equiv \begin{vmatrix} \phi_u & \phi_v & \phi_w \\ \psi_u & \psi_v & \psi_w \\ \chi_u & \chi_v & \chi_w \end{vmatrix}$$

The functions ϕ, ψ, χ are not necessarily analytic but it will be presupposed that

(*a*) the functions ϕ, ψ, χ are of class C'''* in a neighborhood of the origin $(u, v, w) = (0, 0, 0)$;

* We shall say that a single-valued function f of u, v, w is of class C''' if $f(u, v, w)$ and its partial derivatives of orders one, two, and three are continuous in a region in which f is defined.