ON SEPARATED SETS.

BY PROFESSOR W. A. WILSON.

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IN the March number of the BULLETIN appeared a discussion of the definition of Lebesgue integrals given in Pierpont's Theory of Functions of Real Variables, volume II, by Fréchet and the author. The questions there discussed are much simplified if use is made of the *outer* and *inner associated sets* of a point set, concepts due, I believe, to W. H. Young.

These sets are defined in the text mentioned, but for the sake of convenience I shall give their definitions here. They arise at once from the definitions of upper and lower measure of a point set. Let A be the set under consideration. Let it be enclosed in an enumerable set R of rectangular cells, of which the sum of the areas is finite and may be denoted by $\bar{R}$. The minimum of all the possible values of $\bar{R}$ is called the upper measure of A and denoted by $\overline{\text{Meas}}\ A$. Now if a sequence $\{R_n\}$ of the rectangular sets is so chosen that $\lim_{n=\infty} \bar{R}_n = \overline{\text{Meas}}\ A$, their divisor (or set of points common to all of them) will contain A, and will be a measurable point set of measure equal to $\overline{\text{Meas}}\ A$ by the ordinary laws of measurable sets. Such a set is called an outer associated set of A and may be denoted by A_e. There will be an infinity of such sets corresponding to a single A, but for each A_e, $\text{Meas}\ A_e = \overline{\text{Meas}}\ A$ and the sets A_e differ only by a set of measure zero. The inner associated sets are defined as follows. Let A be enclosed in a rectangular cell Q, let $B = Q - A$, and let B_e be an outer associated set of B. Let $A_i = Q - B_e$. Then A_i is contained in A, is measurable and $\text{Meas}\ A_i = \text{Meas}\ Q - \text{Meas}\ B_e = \text{Meas}\ Q - \overline{\text{Meas}}\ B = \underline{\text{Meas}}\ A$, by the definition of lower measure. This set A_i is called an inner associated set of A. Young has also shown that any A_i may be regarded as the union of an enumerable set of complete sets C_n contained in A and such that $\lim_{n=\infty} \text{Meas}\ C_n = \underline{\text{Meas}}\ A$. The importance of these sets is obvious;