A NEW METHOD OF FINDING THE EQUATION OF A RATIONAL PLANE CURVE FROM ITS PARAMETRIC EQUATIONS.

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IN this paper we shall give a new method of deriving the equation of a rational plane curve from its parametric equations.

Let the parametric equations of the rational plane curve R^n of order n be

$$(1)\qquad x_i = a_i t^n + {}^nC_1 b_i t^{n-1} + {}^nC_2 c_i t^{n-2} + \cdots \qquad (i = 0, 1, 2).$$

In particular if $n = 2$, equations (1) become

$$(2)\qquad x_i = a_i t^2 + 2b_i t + c_i \qquad (i = 0, 1, 2),$$

which are the parametric equations of a conic or R^2. The coordinates of the points of the R^2 whose parameters are t_1 and t_2 are $a_i t_1^2 + 2b_i t_1 + c_i$ and $a_i t_2^2 + 2b_i t_2 + c_i$, where $i = 0, 1, 2$. Hence the equation of the line on t_1 and t_2, written out in full, is

$$(3)\qquad \begin{vmatrix} a_0t_1^2 + 2b_0t_1 + c_0 & a_1t_1^2 + 2b_1t_1 + c_1 & a_2t_1^2 + 2b_2t_1 + c_2 \\ a_0t_2^2 + 2b_0t_2 + c_0 & a_1t_2^2 + 2b_1t_2 + c_1 & a_2t_2^2 + 2b_2t_2 + c_2 \\ x_0 & x_1 & x_2 \end{vmatrix} = 0.$$

By subtracting the second row from the first and removing the factor $t_1 - t_2$, equation (3) reduces to

$$(4)\qquad \begin{vmatrix} a_0(t_1 + t_2) + 2b_0 & a_1(t_1 + t_2) + 2b_1 & a_2(t_1 + t_2) + 2b_2 \\ a_0t_2^2 + 2b_0t_2 + c_0 & a_1t_2^2 + 2b_1t_2 + c_1 & a_2t_2^2 + 2b_2t_2 + c_2 \\ x_0 & x_1 & x_2 \end{vmatrix} = 0.$$

The expanded form of (4) is

$$(5)\qquad 2\,|abx|^{*}t_1t_2 + |acx|(t_1 + t_2) + 2\,|bcx| = 0.$$

* By $|abx|$ is meant the three-rowed determinant $\begin{vmatrix} a_0 & b_0 & x_0 \\ a_1 & b_1 & x_1 \\ a_2 & b_2 & x_2 \end{vmatrix}$.