Thus, to the naked eye the courses of two curves, which at a common point have the same tangent and the same radius of curvature, are in the vicinity of that point so nearly identical as to make them appear indistinguishable. The introduction of the notions of axis of aberrancy and osculating parabola serves to magnify the differences between the two curves in such a way as to enable us to distinguish between them. Again, if the two curves also have their osculating parabolas in common, we may judge of their divergence by means of their osculating conics. Thus the notion of osculant serves the differential geometer for the same purpose as does the microscope in the laboratory of the biologist. It magnifies the infinitesimal differences between two different curves sufficiently to cause them to make an emphatic impression upon the mind.

Thus the notions, osculant and penosculant, are the fundamental concepts of differential geometry. The systematic investigation of the magnitudes, loci and envelopes determined by the various classes of osculants and penosculants and the relations which exist between them makes up the whole subject matter of differential geometry. Differential properties of a general curve are merely integral properties of its osculants and penosculants.

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A CERTAIN SYSTEM OF LINEAR PARTIAL DIFFERENTIAL EQUATIONS.

BY DR. H. BATEMAN.

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1. IT is known that if a function $V(x_1, y_1, z_1, t_1; x_2, y_2, z_2, t_2; \cdots; x_n, y_n, z_n, t_n)$ satisfies the system of $\frac{1}{2}n(n+1)$ partial differential equations*

$$(1)\quad \frac{\partial^2 V}{\partial x_p \partial x_q} + \frac{\partial^2 V}{\partial y_p \partial y_q} + \frac{\partial^2 V}{\partial z_p \partial z_q} = \frac{\partial^2 V}{\partial t_p \partial t_q} \quad (p, q = 1, 2, \cdots, n)$$

it becomes a solution of the reduced system of $\frac{1}{2}(n-1)n$

* See for instance H. Bateman, *Messenger of Mathematics*, March, 1914, p. 164.