

The Mathematical Analysis of Electrical and Optical Wave-Motion on the Basis of Maxwell's Equations. By H. BATEMAN, M.A., Ph.D. Cambridge University Press, 1915. vi + 159 pp.

THIS book is intended as an introduction to certain recent developments of Maxwell's electromagnetic theory which are directly connected with the solution of the partial differential equation of wave motion. The higher parts of the theory which are based on the dynamical equations of motion are not considered.

In Chapter I (pages 1-24) the author starts from the fundamental equations for free ether in Maxwell's electromagnetic theory and shows in the first place that solutions of these equations may be obtained by means of solutions of the fundamental wave equation

$$(1) \quad \Omega(u) \equiv \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} = 0,$$

where c is the velocity of light (and is taken to be a constant). A function u of x, y, z, t , satisfying equation (1), is called a wave function. In the second place the author exhibits a class of solutions of the fundamental equations of Maxwell by means of functions α and β (of x, y, z, t) of such a nature that if $F(\alpha, \beta)$ is an arbitrary function of α and β , F satisfies the partial differential equation

$$(2) \quad \left(\frac{\partial F}{\partial x}\right)^2 + \left(\frac{\partial F}{\partial y}\right)^2 + \left(\frac{\partial F}{\partial z}\right)^2 = \frac{1}{c^2} \left(\frac{\partial F}{\partial t}\right)^2.$$

Continuing in Chapter I it is shown that the fundamental Maxwell equations for a material medium may be solved by means of functions u (of x, y, z) satisfying the equation

$$(3) \quad \Delta u + k^2 u \equiv \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} + k^2 u = 0,$$

the quantity k being a constant with respect to x, y, z, t . This equation is obviously satisfied by wave functions of the form

$$u = e^{\pm i k c t f}(x, y, z).$$

In connection with a wave boundary whose equation may be expressed in the form $F(x, y, z, t) = 0$ equation (2) comes again into play and here expresses the fact that the moving wave