

a quadratic equation having integral coefficients. Carmichael has given a different and more exhaustive treatment of these numbers in the *Annals of Mathematics*, 1913. In the present paper Mr. Pierce obtains somewhat similar results for numbers given by the forms $\prod_{i=1}^n (1 \pm \alpha_i^m)$, where the α_i denote algebraic integers defined as the roots of an n th degree equation. The forms of the factors of $\prod_{i=1}^n (1 - \alpha_i^m)$ are determined by use of algebraic number theory, and this perhaps constitutes the most novel result of the work.

6. Lucas has developed the theory of the prime divisors of the functions $U_n = (a^n - b^n)/(a - b)$ and $V_n = a^n + b^n$, where a and b are the roots of a quadratic equation (*American Journal of Mathematics*, volume 1, page 184). Connected with these functions are certain binary forms of degree equal to one half the totient of n , the divisors of which Professor Lehmer has shown to be of the form $2nx \pm 1$. Combining this result with certain results of Mr. Pierce, Professor Lehmer has also obtained a series of numbers the prime factors of which must belong to two such forms, thus restricting notably the character of their divisors.

7. Professor Haskell shows that the condition that a rational fraction whose denominator is the n th power of a quadratic should be rationally integrable, is that the numerator shall be of degree $2(n - 1)$ and that it shall be apolar to the $(n - 1)$ st power of the quadratic factor of the denominator.

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TRANSFORMATION THEOREMS IN THE THEORY OF THE LINEAR VECTOR FUNCTION.

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(Read before the American Mathematical Society, December 31, 1915.)

SINCE the memorable work of Grassmann (1844), the study of the linear transformation has taken various forms, among which are the quaternions of Hamilton, the matrices of Cayley,