A CERTAIN CLASS OF FUNCTIONS CONNECTED WITH FUCHSIAN GROUPS.

BY PROFESSOR ARNOLD EMCH.

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1. CONSIDER a Fuchsian group Γ of linear substitutions

$$V_i \equiv z_i = \frac{\alpha_i z + \beta_i}{\gamma_i z + \delta_i} \qquad (i = 1, 2, 3, \cdots) \tag{1}$$
$$\alpha_i \delta_i - \beta_i \gamma_i = 1,$$

that transform the unit circle into itself, and for which the unit circle is a natural boundary. The index i for which z_i approaches a point of the boundary we denote by ∞, so that $\lim_{i=\infty} (z_i) = e^{i\phi}$, where ϕ may have any value from 0 to 2π. Let $z_0 = z$ represent identity. Denote by $R_0 = R$ the fundamental region in which z lies, and by R_1, R_2, $\cdots$ the regions resulting from R by the substitutions V_i $(i = 1, 2, 3, \cdots)$. Let e_i be the greatest "elongation" of the boundary of R_i, i. e., the maximum distance between two points of the boundary of R_i; then, according to a theorem due to Bricard,* it is possible to circumscribe a circle C_i to the region R_i, such that its radius does not need to be greater than at most $e_i/\sqrt{3}$.

For $i \neq \infty$, the area A_i of R_i, being that of a singly connected region bounded by circular arcs, is finite, so that for the ratio of the area of the circle C_i to that of the region R_i we have

$$1 < \frac{\pi e_i^2}{3A_i} < M \qquad (i = 1, 2, 3, \cdots), \tag{2}$$

where M is a positive finite quantity > 1. But it can be shown that this inequality also exists when $\lim (i) = \infty$, or $\lim_{=\infty} (z_i) = e^{i\phi}$. Hence from (2) we get

$$3\Sigma A_i < \Sigma \pi e_i^2 < 3M\Sigma A_i, \tag{3}$$

* "Théorèmes sur les courbes et les surfaces fermées," *Nouvelles Annales de Mathématique*, 4. ser., vol. 14, pp. 19–25 (January, 1914).