NOTE ON GREEN'S THEOREM.

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1. *Introduction.*—We wish in this paper to extend Green's theorem to apply to relations equivalent to the general linear partial differential equation of the second order in two variables. The relations are written in such a way as not to involve derivatives of the second order, and the theorem is proved without assuming their existence.* This point of view is desirable for the possibility of its application to physics.

2. *Statement of Lemma.*—We shall denote by a_{ij}, a_i, a, arbitrary continuous functions of x and y, the a_{ij} to have also continuous first and second partial derivatives, and the a_i to have continuous first partial derivatives, over a given simply connected region. By b_{ij}, b_i, b, we indicate the coefficients of the linear partial differential expression of the second order which is adjoint to the expression of which the coefficients are a_{ij}, a_i, a. That is, $b_{ij} = a_{ij}$,

$$b_i = 2\frac{\partial a_{i1}}{\partial x} + 2\frac{\partial a_{i2}}{\partial y} - a_i,$$

$$b = \frac{\partial^2 a_{11}}{\partial x^2} + \frac{\partial^2 a_{22}}{\partial y^2} + 2\frac{\partial^2 a_{12}}{\partial x \partial y} - \frac{\partial a_1}{\partial x} - \frac{\partial a_2}{\partial y} + a.$$

* The immediate occasion for the publication of this paper was the appearance of an article by C. W. Oseen, "Über einen Satz von Green und über die Definitionen von Rot und Div," *Rendiconti del Circolo Matematico di Palermo*, vol. 38 (1914), pp. 167–179, which applies this method, by means of the principles of vector analysis, to the special case of Poisson's equation. The general proof is a first step in the detailed working out of a general problem for partial differential equations (see preliminary communication of December, 1913, G. C. Evans, "Green's functions for linear partial differential expressions of the second order, and Green's theorem," BULLETIN, vol. 20, no. 6, March, 1914).

A proof has been given for equations of parabolic type, which may be generalized to the equations here considered. (G. C. Evans, "On the reduction of integro-differential equations," *Transactions Amer. Math. Society*, vol. 15, no. 4, p. 486, Oct., 1914. This paper also gives the literature of the problem). The proof referred to however, by a reduction, makes use of Green's theorem in its usual form; in the present paper, the result is reduced ab initio.