and making a set of five assumptions. It appears that the most general solution, when n is greater than 2, is $\varphi^{-1}f^r\varphi(x)$, where the integer r is prime to n. The case $n = 2$ is discussed separately and a simple algorism is given for reducing all differentiable functions of order 2 to a single type.

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THE LEGENDRE CONDITION FOR A MINIMUM OF A DOUBLE INTEGRAL WITH AN ISOPERIMETRIC CONDITION.

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THE Legendre, or second necessary, condition for a minimum of a double integral, where there is no isoperimetric condition, has been derived by Kobb,* where the equations of the surfaces involved are in parametric form, and by Mason,† where x and y are the independent variables. The analogous condition for the isoperimetric problem has been proved to be sufficient to insure a permanent sign to the second variation,‡ but it has not been proved to be necessary.

In the present paper this condition,

$$h_{pp}(x, y, z, p, q; \lambda)h_{qq}(x, y, z, p, q; \lambda) - h_{pq}^2(x, y, z, p, q; \lambda) \geqq 0,$$

or expressed in parametric form,

$$H_{11}(x, y, z, x_u, x_v, \cdots, z_v; \lambda)H_{22}(x, y, z, x_u, x_v, \cdots, z_v; \lambda) - H_{12}^2(x, y, z, x_u, x_v, \cdots, z_v; \lambda) \geqq 0,$$

is proved to be necessary for either a maximum or a minimum.

Given two functions $f(x, y, z, p, q)$ and $g(x, y, z, p, q)$ and a surface

$$S: \qquad z = z(x, y)$$

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* "Sur les maxima et les minima des intégrales doubles," *Acta Mathematica*, vol. 16 (1892), p. 108.

† "A necessary condition for an extremum of a double integral," BULLETIN, vol. 13 (1907), p. 293.

‡ Kobb, *Acta Mathematica*, vol. 17 (1893), p. 331.