

great pride in using only such methods as he had originated himself, even where simplifications due to others could be introduced to great advantage. Thus, in the second volume of his great work on transformation groups, he not only took scant notice of Mayer's simple and direct derivation of the theory of contact transformations, but ignored completely the notion of the bilinear covariant of a Pfaff differential expression, due to Lipschitz and applied with great success by Frobenius. As S. Kantor has indicated in several papers published in the *Sitzungsberichte* of the Vienna Academy (1901-03), Pfaff's problem and particularly Mayer's theory of contact transformations are greatly simplified by the use of the bilinear covariant; however, Kantor's papers, besides containing a considerable number of errors, are extremely unsystematic and obscure.

In the present work, Engel gives a very clear and simple exposition, based on the bilinear covariant, of the general theory of contact transformations and their invariants, and the application to partial differential equations, and elaborates the generalization, conceived but never worked out by Lie, of the invariant theory to Pfaff expressions in $2n$ variables. The important work of Engel should prove of great value to investigators in this field.

T. H. GRONWALL.

Veränderliche und Funktion. Von M. PASCH. Leipzig, Teubner, 1914. vi + 186 pp.

THIS little book deals primarily with the fundamental ideas at the basis of the theories of variables and functions and with their applications or illustrations by means of some of the most elementary functions. One finds a treatment of such topics as the following: order relations among numbers and the associated ideas, kinds of mathematical proof as illustrated in the foregoing discussion, point sets, sequences, variables and constants, various rational functions and classes of such functions, continuity and uniform continuity, exponential and logarithmic functions, countable sets, etc. The discussion is interspersed with a considerable number of interesting remarks belonging, one may perhaps say, to the philosophy of mathematics.

R. D. CARMICHAEL.