Give to z_2 any value a_2 in S_2, and φ will be analytic in z_1 alone. This holds for every choice of the fixed values assigned to $z_2, \cdots, z_n$. In a similar manner we find φ analytic in each remaining variable alone.

Now apply the theorem of Hartogs* which states that if a function of n complex variables is analytic in each one separately, it is analytic in all n variables taken together. Hence φ is analytic throughout $(S_1, \cdots, S_n)$.

HARVARD UNIVERSITY,
May, 1914.

CONCERNING A CERTAIN TOTALLY DISCONTINUOUS FUNCTION.

BY PROFESSOR K. P. WILLIAMS.

(Read before the American Mathematical Society, October 31, 1914.)

ONE of the most important properties of a continuous function is that it actually assumes every value between any two of its values. It is well known that a function can, however, possess this property without being continuous. An actual example to illustrate this seems to have been first given by Darboux in 1875. A function that is sometimes cited in this connection is due to Mansion.† The function that the latter gives actually takes all values between any two, but is discontinuous at the single point $x = 0$. Functions of this sort can be easily constructed by arbitrarily assigning the values at certain points, according to the function concept of Dirichlet. More interest would therefore attach to such a function if it is given by one and the same expression throughout its region of definition. The function given by Mansion does not, however, possess this property; for it contains the function $E(x)$, defined, as in number theory, as the integer equal to, or next smaller than x.

The purpose of this note is to give a function that takes every value between 0 and 1 inclusive, when x varies over the closed interval (0, 1), but which is discontinuous at every point. This function will, furthermore, be represented by one and the same analytical expression throughout its whole region of definition.

* *Math. Ann.*, vol. 62 (1905), p. 1.

† "Continuité au sens analytique et continuité au sens vulgaire," in *Mathesis*, 1899.