NOTE ON REMOVABLE SINGULARITIES.

BY MR. W. E. MILNE.

IN his dissertation* Kistler proves the following theorem: "*Let* $f(z_1, \cdots, z_n)$ *be analytic throughout the neighborhood of a point* $(a_1, \cdots, a_n)$ *with the exception at most of the points of a finite number of analytic manifolds, each of which is at most* $(2n - 4)$*-dimensional. Then it is analytic in the excepted points also, if properly defined there.*"

A similar theorem is given below. The two theorems differ in two respects. (1) Kistler's theorem requires the excepted points to lie on analytic manifolds, while the present one does not; (2) the present theorem requires that for *every*† pair of variables the excepted locus reduce to isolated points when the remaining $n - 2$ variables are fixed, while Kistler's theorem requires this to hold simply for *one* pair. But it is to be noted that the hypotheses of the present theorem will in general‡ be fulfilled if the singular manifolds are analytic.

The theorem is as follows:

THEOREM. *Let the function* $\varphi(z_1, \cdots, z_n)$ *of* n *complex variables,* $n > 2$, *be analytic in the region* $(S_1, \cdots, S_n)$ *except for the points of a* $(2n - 4)$*-dimensional locus of such character that when any* $n - 2$ *of the variables are given any fixed values in their respective regions, and* z_i *and* z_j *alone vary, then the singularities occur only at isolated points* (a_i, a_j) *in* (S_i, S_j). *Under these conditions* φ *has a limit in the points of the singular locus, and if defined as equal to its limit, will be analytic without exception in* $(S_1, \cdots, S_n)$.

Proof: Let $z_3, \cdots, z_n$ be held fast at $(a_3, \cdots, a_n)$ any point of $(S_3, \cdots, S_n)$, and consider φ as a function of z_1 and z_2. From the hypothesis we see that singularities can occur only at isolated points of (S_1, S_2). But isolated singularities for a function of two complex variables are removable; so φ, if properly defined, will be analytic in z_1 and z_2 throughout (S_1, S_2).

* "Ueber Funktionen von mehreren komplexen Veränderlichen," Basel, 1905. § 7.

† Evidently all that is really necessary is that with each variable z_i it be possible to associate another z_j such that if the remaining $n - 2$ are fixed, the singular points in z_i, z_j are isolated.

‡ I hope shortly to be able to show that the excepted cases can always be avoided by a suitable linear transformation of the independent variables, and hence that the words "in general" can be replaced by "always."