

TIME AS A FOURTH DIMENSION.

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WHO first conceived the idea of time as a fourth dimension? In view of the recent development of the theory of relativity the answer to this question is of some interest to both mathematician and physicist. With the definite formulation of our ideas concerning space of n dimensions during the latter part of the last century the conception occurred, doubtless, to many people. But, so far as I am aware, writers who discuss the subject have invariably given the credit to Lagrange (1736–1813), and have referred to a paragraph in either the first* (1797, Loria† and Enriques‡) or second§ (1813, Dühring|| and Sommerville¶) editions of his *Théorie des Fonctions analytiques*.** This paragraph, the first of the section “Application de la théorie des fonctions à la mécanique,” is as follows:††

“Nous allons employer la théorie des fonctions dans la mécanique. Ici les fonctions se rapportent essentiellement au temps, que nous désignerons toujours par t ; et comme la position d’un point dans l’espace dépend de trois coordonnées rectangulaires x , y , z , ces coordonnées, dans les problèmes de mécaniques, seront censées être fonctions de t . Ainsi on peut regarder la mécanique comme une géométrie à quatre dimensions, et l’analyse mécanique comme une extension de l’analyse géométrique.”

* Paris, Prairial, an V [= 1797], p. 223.

† G. Loria, *Die hauptsächlichsten Theorien der Geometrie . . . ins Deutsche übertragen von F. Schütte*, Leipzig, 1888, pp. 116–117; also *Il Passato ed il Presente delle principali Teorie geometriche*, terza ed., Torino, 1907, p. 327.

‡ F. Enriques, *Encyclopédie des Sciences Math.*, tome III, vol. 1, p. 71 (1911); also *Encyklopädie d. Math. Wiss.*, Bd. III, p. 63 (1907).

§ Paris, 1813, p. 311.

|| E. Dühring, *Kritische Geschichte der allgemeinen Principien der Mechanik*. Leipzig, 1877, p. 562f.

¶ D. M. Y. Sommerville, *Bibliography of Non-Euclidean Geometry*, London, 1911, p. 17.

** There are four other editions of this work: A German translation of the first by J. P. Grison, Berlin, 1798–99; a German edition, with notes, of the second by A. L. Crelle in Bd. I of *Lagrange’s Mathematische Werke*, Berlin, 1823; a fourth French edition reprinted from the second in *Lagrange’s Œuvres*, tome IX (1881); the third French edition (best), edited by J. A. Serret, Paris, 1847.

†† I quote from the second edition. The only difference of the paragraph from that of the first edition is in the first sentence.