and, as for the summability (Ck) of the series $u_0 + u_1 + \cdots + u_n + \cdots$ it is necessary that*

$$\lim_{n=\infty} \frac{u_n}{n^k} = 0,$$

it follows that, for any $k < \frac{1}{4}$, we obtain a Fourier series which is not summable (Ck) for any value of x by selecting a ν such that $1 - 2\nu > 4k$. By a suitable modification of Riemann's example, we may construct a Fourier series with the corresponding property for any $k < \frac{1}{2}$; for $1 > k \geqq \frac{1}{2}$, I have not been able to decide whether the theorem is true for all integrable (and not only absolutely integrable) functions or not.†

CHICAGO, ILL.,
February 3, 1913.

NOTE ON PIERPONT'S THEORY OF FUNCTIONS.

IN a review, written some years ago, of Pierpont's Theory of Functions of Real Variables, I made the following incorrect statement with regard to the possibility of reversing the order of differentiation of a function $f(x, y)$:‡

"Under the assumption that f_x' exists on $y = b$, f_y' on $x = a$, and that one of them is approached uniformly, it follows as a corollary to the theorem of Moore mentioned above, that the second derivatives f_{xy}'', f_{yx}'' exist at (a, b) and are equal."

The assumptions should be that f_x' exists on $x = a$, f_y' on $y = b$, and that the derivative for x at $x = a$ of the quotient $f(x, y)/(y - b)$ is approached uniformly for values of y different from b. These are the hypotheses, in different words, which Professor E. H. Moore uses in the Lectures referred to on page 124 of the review, and which I intended to reproduce.

I am indebted for this correction to Mr. G. A. Pfeiffer. In a recent letter to me he cited the example $f = xy(x^2 - y^2)/(x^2 + y^2)$ with the agreement that f shall be zero for $x = y = 0$, which

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* S. Chapman, l. c., p. 379.

† For $k \geqq 1$, the theorem holds for any integrable function; see for the case $k = 1$ (the theorem holds a fortiori for $k > 1$) L. Fejér, "Untersuchungen über trigonometrische Reihen," *Math. Annalen*, vol. 58 (1904), pp. 51–69.

‡ BULLETIN, vol. 13 (1906), page 125.