ON CLOSED CONTINUOUS CURVES.

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(Read before the Southwestern Section of the American Mathematical Society, November 30, 1912.)

1. In a paper soon to be published it will be proved that in every closed convex curve which is analytic throughout at least one square may be inscribed. Conversely, if in a Cartesian plane an arbitrary square is given, the problem is to find the parametric equations of any convex curve through the vertices of the given square. It is the purpose of this paper to establish these equations.

2. First assume the square $A_1A_2A_3A_4$ symmetric with respect to the x- and y-axes and on the circle

$$x = \frac{a}{2}\sqrt{2}\cos\frac{2\pi t}{w}, \qquad y = \frac{a}{2}\sqrt{2}\sin\frac{2\pi t}{w}, \tag{1}$$

where t is the parameter and w the period. The coordinates of A_1, A_2, A_3, A_4 are in the same order $(a/2, a/2)$; $(-a/2, a/2)$; $(-a/2, -a/2)$; $(a/2, -a/2)$, and the corresponding parameters $t_k = (2k+1)\cdot w/8$, $(k = 0, 1, 2, 3)$. Designating by $\phi(t)$, $\psi(t)$ two uniform continuous functions for all values of t, and with the same period w, then

$$\begin{aligned} x &= \frac{a}{2}\sqrt{2}\cos\frac{2\pi t}{w} + \lambda\sin\left(\frac{2\pi t}{w} - \frac{\pi}{4}\right)\sin\left(\frac{2\pi t}{w} - \frac{3\pi}{4}\right)\phi(t), \\ y &= \frac{a}{2}\sqrt{2}\sin\frac{2\pi t}{w} + \mu\sin\left(\frac{2\pi t}{w} - \frac{\pi}{4}\right)\sin\left(\frac{2\pi t}{w} - \frac{3\pi}{4}\right)\psi(t),^* \end{aligned} \tag{2}$$

where λ and μ are arbitrary constants, represent a closed curve through the vertices of the square. Now any closed continuous curve† may be represented parametrically by

$$x = F(t), \qquad y = G(t), \tag{3}$$

* In this expression it is sufficient to take the product of two sines as indicated, not as in the abstract of the paper which appeared in the BULLETIN of February, pp. 221–222, where a product of four sines was introduced.

† Osgood: Lehrbuch der Funktionentheorie, vol. 1, 2d ed., pp. 146–150.