CONCERNING THE PROPERTY Δ OF A CLASS OF FUNCTIONS.

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IN the memoir* entitled "Introduction to a Form of General Analysis" E. H. Moore has studied real-valued functions and classes of real-valued functions of a general variable. He has given generalizations of numerous well-known theorems, has exhibited many new phenomena, and has indicated the important rôle this type of analysis is likely to play. The theory relates to properties of classes of functions. Some of these properties we define in the immediate sequel. In order to make application of the theory to a particular class of functions one must know whether or not the class possesses certain of the above-mentioned properties. One of the more difficult of these properties to study is the property Δ defined below (cf. the above memoir, § 79). The present paper establishes some theorems which are likely to be of service in this connection and which may be of interest in themselves. The page and section references of the sequel are all to Moore's memoir cited above.

A class $\mathfrak{M}$ of functions μ on a general range $\mathfrak{P}$ (page 4; § 4) is said to be *linear* (§ 14) in case every function of the form $a_1\mu_1 + a_2\mu_2$, where a_1, a_2 are arbitrary real numbers and μ_1, μ_2 are arbitrary functions of $\mathfrak{M}$, is of $\mathfrak{M}$.

A class $\mathfrak{M}$ is said to have the *dominance* property D if for every sequence $\{\mu_n\}$ of functions of $\mathfrak{M}$ there is a sequence $\{a_n\}$ of real numbers and a function μ_0 of $\mathfrak{M}$ such that for every n and for every element p of the range $\mathfrak{P}$

$$|\mu_n(p)| \leqq a_n|\mu_0(p)|.$$

A *development*† Δ (§ 75) of a class $\mathfrak{P}$ of elements p is a system

$$((\mathfrak{P}^{ml})) \quad (m = 1, 2, 3, \cdots; \quad l = 1, 2, 3, \cdots, l_m)$$

* See New Haven Mathematical Colloquium, New Haven, Yale University Press, 1910.

† The following is a concrete example. Let $\mathfrak{P}$ be the class of all real numbers p such that $0 \leqq p \leqq 1$. Stage m of a development of $\mathfrak{P}$ is a set of $m + 1$ overlapping intervals of $\mathfrak{P}$. Thus for each m, $l_m = m + 1$, and for every m, l the class $\mathfrak{P}^{ml}$ is the interval $((l - 2)/m, l/m)$ where $-1/m$ is taken as 0 and $(m + 1)/m$ is taken as unity (§ 66a). A representative system for this development is the system $((r^{ml}))$ of numbers such that $r^{ml} = (l - 1)/m$ (§ 66a).