

furthermore let

$$|f_i(x, t, y)| \leq M \quad (i = 1, 2).$$

Then if ρ satisfies the conditions

$$\rho \leq \alpha, \quad \rho \leq \frac{\beta}{M}, \quad \rho \leq \frac{\gamma}{2AM},$$

there exists, in the interval $|x - x_0| \leq \rho$, one and only one continuous solution $y(x)$ of the integral equation (1).

The proof is entirely similar to that of Theorem I.

CORNELL UNIVERSITY,
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A THEOREM ON ASYMPTOTIC SERIES.

BY MR. VINCENT C. POOR.

THEOREM: *If $f(z)$ is not holomorphic at $z = 0$, but is formally developable into a Maclaurin series, and if w is asymptotic to $a_1/z + a_2/z^2 + \dots$ (written: $w \sim a_1/z + a_2/z^2 + \dots$), then $f(w)$ has an asymptotic representation.**

To prove this theorem take $f(z)$ in the form

$$(1) \quad f(z) = f(0) + f'(0)z + \frac{f''(0)}{2!}z^2 + \dots \\ + \frac{f^{(n)}(0)z^n}{n!} + \int_0^z f^{(n+1)}(t) \cdot \frac{(z-t)^n}{n!} dt.$$

Since

$$w \sim \frac{a_1}{z} + \frac{a_2}{z^2} + \dots$$

w may be written

$$(2) \quad w = \frac{a_1}{z} + \frac{a_2}{z^2} + \dots + \frac{a_n + \epsilon_{1nz}}{z^n},$$

where, according to the Poincaré definition† for an asymptotic

* This theorem is the "résultat préalablement obtenu" referred to in Professor Ford's paper in the *Bulletin* of the French Society for 1911. See *Bulletin Société math. de France*, vol. 40 (1912), fascicule 1 under "Erratum du Tome XXXIX."

† Poincaré, *Acta Mathematica*, vol. 8 (1886), p. 296.