

## SHORTER NOTICES.

*Elliptische Funktionen.* Von Professor Dr. KARL BOEHM.  
Zweiter Teil. Götschen (Sammlung Schubert LXI), Leipzig,  
1910. vii+180 pp. M. 5.

THIS little book is complete in itself and independent of the first volume.\* It is devoted entirely to the older theory, that is, the development of the properties of elliptic integrals and the inversion problem.

In the first chapter we have a rather thorough discussion of hyperelliptic integrals. There is, first, the reduction of the general hyperelliptic integral to the three possible types and, second, the classification of these integrals according to deficiency (Geschlecht). The elliptic integrals are thus introduced as a particular class of hyperelliptic integrals (cf. Picard, *Traité*, Tome I).

The second chapter deals with the behavior of the elliptic integral of the first kind in the complex plane and upon the proper Riemann surface, leading to the existence and meaning of the two period moduli.

The third chapter is devoted to the inversion of the elliptic integral of the first kind. The point of departure is the theorem concerning the existence of the solution of the differential equation  $dz/du = F(z)$ , where  $F(z)$  is an analytic function of  $z$ .

A careful proof of the uniqueness of the inverse function is given; the final conclusion being that this function is meromorphic in the entire  $u$  plane. The results of the preceding chapter enable the author to add that the inverse function is doubly periodic and of the second order. In this way, then, we are led to the elliptic function of the second order. The chapter closes with the theorem:

The most general elliptic integral can be expressed as the integral of an elliptic function  $\phi(u)$ , where  $u$  is the elliptic integral of first kind belonging to the same irrationality as appears in the general integral.

The fourth chapter considers the integral of the first kind and its inversion when the branch points are real. There is

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\* Reviewed in the BULLETIN for January, 1911, vol. 17, p. 202.