a symmetric or under an alternating group. It may also be observed that if G is an imprimitive group of transformations of Sylow subgroups of order p^m, then G cannot involve an invariant subgroup whose order is of the form p^α, since its degree is of the form $1 + kp$. If the systems of imprimitivity of G are transformed according to a group involving smaller Sylow subgroups of the form p^α than those contained in G, it results from the theorem proved above that G contains other systems of imprimitivity which are transformed according to Sylow subgroups whose orders of the form p^α are equal to the orders of the corresponding Sylow subgroups of G. Hence the theorem:

If G is an imprimitive group of transformations of Sylow subgroups of order p^m and involves Sylow subgroups of order p^α, then G must have systems of imprimitivity which are transformed according to a group involving Sylow subgroups of order p^α.

UNIVERSITY OF CHICAGO.

THEOREMS ON FUNCTIONAL EQUATIONS.

BY MR. A. R. SCHWEITZER.

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1. IN the BULLETIN, volume 18 (1912), page 300, we have referred to Abel, *Crelle's Journal*, volume 2 (1827), page 389, in relation to the equation

$$\psi(x) - \psi(y) = \Omega^{-1}\{\phi(x, y)\}. \tag{1}$$

This reference suggests

$$\psi(x) - \psi(y) = \Omega^{-1}\{x\phi(y) - y\phi(x)\} \tag{2}$$

as a correlative of the functional equation* discussed by Abel, l. c., namely,

$$\psi(x) + \psi(y) = \Omega^{-1}\{x\phi(y) + y\phi(x)\}. \tag{2'}$$

Further special cases of the equation (1) are obtained by considering the generalizations of equation (2′) by Lottner,

* Cf. Cayley, Mathematical Papers, vol. IV, pp. 5–6.