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SURFACES OF REVOLUTION OF MINIMUM
RESISTANCE.

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THE problem of finding the surface of revolution of minimum resistance may be thought of as the oldest problem of the calculus of variations. A first solution was given by Newton* in 1686. It has since been considered by L'Hospital, August, Silvabelle, Kneser, and others. The results obtained by these writers are based on the Newtonian law of resistance, which states that the resistance R is given by the formula

$$R = f \cdot \sin^2 \alpha,$$

where f is the force and α the angle which the line of force makes with the tangent to the surface at the point of application. However, physical experiment does not always verify this law. Especially does it fail† when the angle α is small.

As a result of this, several different laws of resistance have been given; some being derived mathematically, others being stated as verifying experiment. Among these the laws of von Lössl,‡ Duchemin,§ and Kirchhoff|| have received the greater notice. They are given by the following formulas:

$$R = f \sin \alpha, \quad R = f \frac{2 \sin \alpha}{1 + \sin^2 \alpha}, \quad R = f \frac{(4 + \pi) \sin \alpha}{4 + \pi \sin \alpha}.$$

* See *Principia Philosophiæ Naturalis*, II; Sect. VII, Prop. xxxiv, Scholium.

† For an account of the various physical causes underlying this, see *Encyklopädie der mathematischen Wissenschaften*, IV, 17, §§ 4, 5, 6.

‡ F. v. Lössl, *Die Luftwiderstandsgesetze*, Wien, 1896, p. 96.

§ Duchemin, *Experimentaluntersuchungen über den Widerstand der Flüssigkeiten*, Braunschweig, 1844, p. 101. This law has been verified by Langley, *Experiments in Aërodynamics*, Washington, 1891, p. 101.

|| G. Kirchhoff, *Journal für Mathematik*, vol. 70 (1869).