

satisfies the conditions stated above for all values of x , it was shown that a precisely analogous theorem holds for the approximation of $f(x)$ by a trigonometric sum of order n or lower, this result being obtainable as a consequence of the preceding. It is now shown that decided simplification in the proof of both theorems may be effected by proving the second directly (this had been done only for $k = 1$) and deducing the first from it.

This method has the further advantage that the numerical constants involved can be computed more conveniently. For example, if $f(x)$ satisfies the condition

$$|f(x_2) - f(x_1)| \leq |x_2 - x_1|$$

in the closed interval $(0, 1)$, it can be approximately represented in this interval by a polynomial of degree n or lower, with an error which never exceeds $3/n$, for all positive integral values of n . The same line of investigation leads to results in the theory of Fourier's series.

13. There is a theorem that the perpendiculars let fall from the incenters of three out of four lines of given direction upon the remaining line touch a circle. In Dr. Hodgson's paper a circle is obtained for any even number of lines, beginning with four. If we take this circle for any $2n$ out of $2n + 1$ lines, the $2n + 1$ circles touch a line. The question of the reversal of direction of one or more of $2n$ lines is then taken up, and this is followed by the consideration of the configuration of circles arising from four, five, and six lines.

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ON THE FOUNDATIONS OF THE THEORY OF LINEAR INTEGRAL EQUATIONS.*

BY PROFESSOR E. H. MOORE.

1. *The Analogous Systems of Linear Equations.*

THE theory of linear integral equations, mathematically considered, has its taproot in the classical analogies between

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