where the large letters are the cofactors of the corresponding small letters in Δ_n. It will be noticed that each member of (6) is of degree $3n$, as it should be.

UNIVERSITY OF WASHINGTON,
March, 1911.

NOTE ON THE MAXIMAL CYCLIC SUBGROUPS OF A GROUP OF ORDER p^m.

BY PROFESSOR G. A. MILLER.

(Read before the San Francisco Section of the American Mathematical Society, October 28, 1911.)

IF H is any non-invariant subgroup of a group G of order p^m, p being any prime number, it is well known that H is transformed into itself by at least one of its conjugates under G and hence by operators which are not contained in H.* If H is cyclic and not contained in a larger cyclic subgroup of G, it is said to be a *maximal cyclic subgroup of* G. In what follows we shall establish the

THEOREM: *A necessary and sufficient condition that every maximal cyclic subgroup of order p^α in a group G of order p^m, $m > 3$, is transformed into itself by no more than $p^{\alpha+1}$ operators of G is that G contains one and only one cyclic subgroup of order p^{m-1}.*

If we combine with this theorem some well-known properties of the groups of order p^m which contain operators of order p^{m-1}, it results that there are only three non-cyclic groups of order p^m which have the property that each of their maximal cyclic subgroups of order p^α is transformed into itself by only $p^{\alpha+1}$ operators of the group. These three groups are the three non-cyclic groups of order 2^m which involve one and only one cyclic subgroup of order 2^{m-1}.

To prove the theorem in question, we shall assume that G does not involve any operator of order p^{m-1}, since the groups of order p^m which contain operators of order p^{m-1} are so well known. We shall also assume in what follows that G satisfies the condition that each one of its maximal cyclic subgroups of order p^α is transformed into itself by exactly $p^{\alpha+1}$ operators of G, p^α being the order of any one of the maximal cyclic subgroup of G.

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* Cf. *American Journal of Mathematics*, vol. 23 (1901), p. 173.