equations (2) it follows that

$$\frac{\partial f_1}{\partial y_1}\frac{\Delta y_1}{\Delta x_1}+\frac{\partial f_1}{\partial y_2}\frac{\Delta y_2}{\Delta x_1}+\cdots+\frac{\partial f_1}{\partial y_n}\frac{\Delta y_n}{\Delta x_1}+\frac{\partial f_1}{\partial x_1}=0,$$
$$\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot$$
$$\frac{\partial f_n}{\partial y_1}\frac{\Delta y_1}{\Delta x_1}+\frac{\partial f_n}{\partial y_2}\frac{\Delta y_2}{\Delta x_1}+\cdots+\frac{\partial f_n}{\partial y_n}\frac{\Delta y_n}{\Delta x_1}+\frac{\partial f_n}{\partial x_1}=0,$$

where the arguments of the derivatives $\partial f_i/\partial x_1$ have the form $x+\theta_i'\Delta x; y+\Delta y$. Hence as Δx_1 approaches zero the quotients $\Delta y_i/\Delta x_1$ approach limits $\partial y_i/\partial x_1$ which satisfy the equations

$$\frac{\partial f_1}{\partial y_1}\frac{\partial y_1}{\partial x_1}+\frac{\partial f_1}{\partial y_2}\frac{\partial y_2}{\partial x_1}+\cdots+\frac{\partial f_1}{\partial y_n}\frac{\partial y_n}{\partial x_1}+\frac{\partial f_1}{\partial x_1}=0,$$
$$(3)\qquad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot$$
$$\frac{\partial f_n}{\partial y_1}\frac{\partial y_1}{\partial x_1}+\frac{\partial f_n}{\partial y_2}\frac{\partial y_2}{\partial x_1}+\cdots+\frac{\partial f_n}{\partial y_n}\frac{\partial y_n}{\partial x_1}+\frac{\partial f_n}{\partial x_1}=0,$$

where the arguments of the derivatives of f are now $(x; y)$. A similar consideration shows the existence of the first derivatives with respect to the variables $x_2, x_3, \cdots, x_m$. The existence of the higher derivatives follows from the observation that the solutions of equations (3) are differentiable $n-1$ times with respect to the variables x on account of the assumption that the functions f are differentiable n times.

ON A SET OF KERNELS WHOSE DETERMINANTS FORM A STURMIAN SEQUENCE.

BY MR. H. BATEMAN, M.A.

WEYL * has recently given a theorem which states that if a kernel

$$k_n(s,t)=\sum_{p,q=1}^{n}k_{pq}\Phi_p(s)\Phi_q(t)\qquad(k_{pq}=k_{qp})$$

is formed from n functions $\Phi_p(s)$ whose squares are integrable in the interval (0, 1), then the smallest positive root of the

* *Göttinger Nachrichten*, 1911, Heft 2, p. 110.