A NEW PROOF OF THE EXISTENCE THEOREM FOR IMPLICIT FUNCTIONS.

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THE theorem with which this paper has to do is the one which states the existence of a set of functions

$$y_i = y_i(x_1, x_2, \cdots, x_m) \qquad (i = 1, 2, \cdots, n)$$

which satisfy a system of equations of the form

$$(1) \qquad f_i(x_1, x_2, \cdots, x_m; y_1, y_2, \cdots, y_n) = 0 \quad (i = 1, 2, \cdots, n).$$

For the case in which the functions f are only assumed to be continuous and to have continuous first derivatives, the proof seems to have been originally given by Dini.* His method is to show the existence of a solution of a single equation, and then to extend his result by mathematical induction to a system of the form given above, a plan which has been followed, with only slight alterations and improvements in form, by most writers on the theory of functions of a real variable. In a more recent paper† Goursat has applied a method of successive approximation which enabled him to do away with the assumption of the existence of the derivatives of the functions f with respect to the independent variables x.

One can hardly be dissatisfied with either of these methods of attack. It is true that when the theorem is stated as precisely as in the following paragraphs, the determination of the neighborhoods at the stage when the induction must be made is rather inelegant, but the difficulties encountered are not serious. The introduction of successive approximations is an interesting step, though it does not simplify the situation and indeed does not add generality with regard to the assumptions on the functions f. The method of Dini can in fact, by only a slight modification, be made to apply to cases where the functions do not have derivatives with respect to the variables x.

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* Lezioni di Analisi infinitesimale, vol. 1, chap. 13. For historical remarks, see Osgood, Encyclopädie der mathematischen Wissenschaften, II, B 1, § 44 and footnote 30.

† *Bulletin de la Société mathématique*, vol. 31 (1903), page 185.