where K'' is a positive constant, and consequently $f(x)/x^r \rightarrow 0$ as $x \rightarrow \infty$, which was what we desired to prove.

In conclusion, we may remark that the theorem may be stated as one of pure integral calculus, without reference to the theory of summability of integrals. Putting $f(x) = \phi(x)x^r$, the theorem thus becomes:

If $\phi(x)$ is uniformly continuous over the infinite interval $x \geqq k > 0$, then the convergence to a limit, as $x \rightarrow \infty$, of the integral

$$\int_0^x \phi(\beta)\beta^r \left(1 - \frac{\beta}{x}\right)^r d\beta$$

requires that $\phi(x)$ shall $\rightarrow 0$ as $x \rightarrow \infty$.

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IRREDUCIBLE HOMOGENEOUS LINEAR GROUPS OF ORDER p^m AND DEGREE p OR p^2.

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(Read before the American Mathematical Society, February 25, 1911.)

No group all of whose non-invariant commutators give invariant commutators besides identity can be simply isomorphic with irreducible groups of different degrees. This category includes all groups of order p^m (p a prime) and classes one, two, and three. Moreover no group of order p^m can be simply isomorphic with irreducible groups of just two different degrees.*

A consideration of these facts gives rise to the query as to whether any group of order p^m can be simply isomorphic with irreducible groups of different degrees, and it is the purpose of this note to answer this question for certain special cases.

In the first place, if G is an irreducible group of order p^m and degree p, it cannot be simply isomorphic with an irreducible group of any other degree, since it contains an abelian subgroup of index† p, and since a group of order p^m with an abelian sub-

* BULLETIN, vol. 14 (1908), pp. 328, 329.

† *Transactions Amer. Math. Society*, vol. 7 (1906), p. 68. We shall have occasion to make use of the fact, established here, that in an irreducible group of order p^m and degree p, the substitutions commutative with a substitution that gives an invariant commutator besides identity form an abelian subgroup.