matical literature sadly reminds us; but one who like Sturm can seize on the important and simple modifications of a given problem has certainly one of the most essential elements of mathematical greatness.

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A SENSUOUS REPRESENTATION OF PATHS THAT LEAD FROM THE INSIDE TO THE OUTSIDE OF AN ORDINARY SPHERE IN POINT SPACE OF FOUR DIMENSIONS WITHOUT PENETRATING THE SURFACE OF THE SPHERE.

BY PROFESSOR C. J. KEYSER.

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THE logical or analytic existence of such paths—their existence in and for thought as distinguished from intuition or imagination—has been long familiar to every one, and may be made evident even to a freshman, so simple is the sufficient algebraic argument. But all efforts to envisage the paths are defeated completely.

It is the purpose of this note to show how the existence of the paths may be made evident to the intuition and even to the senses of sight and touch. The purpose is achieved by a simple transformation correlating the points of 4-space S_4 with the spheres of ordinary space S_3, including all spheres of real center and pure imaginary radius. In this way unintuitable situations in S_4, like that presented by the paths in question, are represented by intuitable analytic equivalents in S_3, and these equivalents may be rendered sensible by easily constructible physical models.

The simplest possible correlation of the kind in question is that in which the point (x, y, z, w) of S_4 and the sphere (of S_3) having (x, y, z) for center and $\sqrt{w}$ for radius shall be a pair of correspondents.

The representative in S_3 of a lineoid (an ordinary 3-space) $Ax + By + Cz + Dw + E = 0$ of S_4 is a linear complex of spheres such that, if (x_1, y_1, z_1, w_1) be a point of the lineoid, the