if the integral is taken in the ordinary or Riemannian sense, we must presuppose the integrability of the expressions $W(x) \cdot W_{n,r}(x) / [W_n(x)]^2$, but this is not necessary if the Lebesgue integral is used, provided, at least, the above expression is finite in $[a, b]$. The condition $W_n(x) \neq 0$ in $[a, b]$ may also be removed in certain cases, as when $W_n(x)$ vanishes at only a finite number of points, $x_0, x_1, \cdots, x_m$ in $[a, b]$ so that $\lim_{x=x_i} W_r(x)/W_n(x)$ exists and is finite for $\binom{r=0, 1, \ldots, n-1}{i=0, 1, \ldots, m}$. It seems unlikely, however, that formula (8) can be so extended as to give new criteria for linear dependence.

In conclusion we note that formulas (3), or (3′), and (8) taken together express G in terms of W in such a way as to show that $G = 0$ when W vanishes, under the restrictions named. We thus have what may be regarded as the converse of (2). Similarly (8) and (2) express D in terms of G.

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NOTE ON INTEGRATION OF SERIES BY LEBESGUE INTEGRALS.

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LEBESGUE has shown in his work on integration that if a limited function, $f(x) \geqq 0$, is measurable for a measurable field A, it is "summable," or possesses a Lebesgue integral, and the value of this integral is the measure of the ordinate set Y, whose points are defined by the conditions: x in A, $0 \leqq y \leqq f(x)$. The converse is also true; that is, if Y is measurable, $f(x)$ is measurable and

$$\text{meas } Y = \int_A f(x) dA.$$

A proof of this may be found in Schoenfliess, *Jahresbericht der Deutschen Mathematiker-Vereinigung*, Ergänzungsband II, part II, page 320.

It is the purpose of this note to show how by use of this idea the proof of Lebesgue's theorem on termwise integration of series can be greatly simplified and reduced to elementary theorems on point aggregates. The theorem in question is proved in Hobson's Theory of Functions of a Real Variable,