ON THE SADDLEPOINT IN THE THEORY OF MAXIMA AND MINIMA AND IN THE CALCULUS OF VARIATIONS.

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Introduction.

LAGRANGE has shown that the problem of determining a function $y(x)$ which satisfies the boundary conditions

$$(1) \qquad y(0) = y(1) = 0$$

and the integral condition

$$(2) \qquad \int_0^1 g(x, y, y')dx = 0$$

and which minimizes the integral

$$(3) \qquad \int_0^1 f(x, y, y')dx$$

is equivalent, as far as the first variation is concerned, to the problem of minimizing the integral

$$(4) \qquad \int_0^1 (f + \lambda g)dx,$$

the function being subject to no isoperimetric condition. The two constants of integration and the isoperimetric constant λ of the Lagrange differential equation

$$\frac{d}{dx}(f_{y'} + \lambda g_{y'}) + f_y + \lambda g_y = 0$$

which furnishes the solution are determined from the conditions (1) and (2). On the other hand it is possible to consider the problem of minimizing the integral (4) subject only to the boundary condition (1), in which case the minimum is obviously a function of the parameter λ. The determination of that value of λ which maximizes this minimum is a saddlepoint problem. The methods of this paper suffice to show that the first necessary condition for a solution $y(x)$ of this problem is identical with that for a solution of the foregoing isoperimetric problem.