ON THE FACTORIZATION OF INTEGRAL FUNCTIONS WITH p-ADIC COEFFICIENTS.

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1. IF $F(x)$ is an integral function of degree n with integral p-adic coefficients, then for any integer k we have a congruence

$$(1)\qquad F(x) \equiv F^{(k)}(x) = F_0(x) + pF_1(x) + p^2F_2(x) + \cdots + p^kF_k(x) \pmod{p^{k+1}},$$

in which each $F_i(x)$ is an integral function of degree $\leqq n$ with coefficients belonging to the set $0, 1, \cdots, p-1$. The function $F^{(k)}(x)$ is called the convergent of rank k of $F(x)$. If

$$(2)\qquad F(x) = f(x)\cdot g(x) \qquad (p),$$

in which the factors are integral functions with integral p-adic coefficients, then for any integer k we obviously have

$$(3)\qquad F^{(k)}(x) \equiv f^{(k)}(x)\cdot g^{(k)}(x) \pmod{p^{k+1}}.$$

The following converse theorem plays a fundamental rôle in Hensel's new theory of algebraic numbers:* *If*

$$(4)\qquad F(x) \equiv f_0(x)\cdot g_0(x) \pmod{p^{s+1}}$$

for † $s + 1 > 2\rho$, *where* ρ *is the order of the resultant* R *of* $f_0(x)$ *and* $g_0(x)$, *then* $F(x)$ *is the product* (2) *of two integral functions with integral p-adic coefficients whose convergents of rank* $s - \rho$ *are* $f_0(x)$ *and* $g_0(x)$.

Hensel's proof is in effect a process to construct the successive convergents of $f(x)$ and $g(x)$. Each step of the process requires the solution of a linear equation in two unknowns with p-adic coefficients. The object of this note is to furnish a decidedly simpler process, which dispenses with these linear

* Hensel, Theorie der algebraischen Zahlen, Leipzig, Teubner, 1908, p. 71.

† This condition is satisfied if $s = \delta$, where δ is the order of the discriminant of $F(x)$. Hence we obtain as a corollary the theorem of Hensel, page 68.