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NOTE ON IMPLICIT FUNCTIONS DEFINED BY TWO EQUATIONS WHEN THE FUNCTIONAL DETERMINANT VANISHES.

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1. *Introduction.* Consider the two equations

$$(1)\quad f(x_1, x_2, \cdots, x_n; y, z) = 0, \quad g(x_1, x_2, \cdots, x_n; y, z) = 0,$$

and suppose a single point solution

$$(2)\qquad x_1 = a_1, x_2 = a_2, \cdots, x_n = a_n; y = b, z = c,$$

is known. Under certain well-known conditions, of which one is the non-vanishing of the functional determinant $\partial(f, g)/\partial(y, z)$ at the point in question, we may affirm that equations (1) define uniquely the functions

$$(3)\qquad y = \phi(x_1, x_2, \cdots, x_n), \quad z = \psi(x_1, x_2, \cdots, x_n)$$

in the neighborhood of the system of values (2). In general if the functional determinant vanishes, the functions (3) are multiple valued.* There are, however, certain exceptional cases in which the determination of y and z as functions $x_1, x_2, \cdots, x_n$ is unique although the functional determinant vanishes. It is proposed in this paper to examine briefly some of these exceptional cases.

In a certain trivial way the corresponding exceptional cases exist also when we consider a single equation defining one dependent variable. Suppose an equation for the determination of y as a function of x has the form

$$(4)\qquad f(x, y) \equiv x f_1(x, y) = 0.$$

* The case of analytic functions was treated by Professor G. A. Bliss in his Princeton Colloquium Lectures (1909). These lectures have not yet been published.