MULTIPLY PERIODIC FUNCTIONS.

An Introduction to the Theory of Multiply Periodic Functions. By H. F. BAKER, SC.D., F.R.S., Fellow of St. John's College and Lecturer in Mathematics in the University of Cambridge. Cambridge University Press, 1907. Royal 8vo. xv + 335 pp.

THIS is a highly interesting and suggestive contribution to a field which has engaged the attention of numerous mathematicians since the time of Abel. Except for the first chapter, the present work has little in common with other treatises relating to the same subject, while a considerable portion of the material is drawn from the author's own investigations.

The book is divided into two parts, the first dealing with hyperelliptic functions of two variables, the second with periodic functions of n variables with reference to the fundamental problem of their connection with the theory of algebraic functions and their expression in terms of the Riemann theta functions.

"The first part is centered round some remarkable differential equations satisfied by the functions, which appear to be equally illuminative both of the analytic and the geometric aspects of the theory; it was, in fact, to explain this that the book was originally entered upon." Chapter I is introductory and is chiefly concerned with a deduction of the fundamental formulas connected with a Riemann surface of two sheets and six branch points. It contains a brief and condensed account of the hyperelliptic integrals of the first, second, and third kinds, and their behavior on the surface. After developing the properties of the theta functions, a notable departure is made from the usual treatment. Following closely the analogy of the Weierstrassian theory of elliptic functions, a single theta function is retained out of the sixteen with half-integer characteristics. This is multiplied by an exponential factor and the product regarded as a function of the unnormalized integrals u_1, u_2 of the first kind and of their homogeneous table of moduli. The function so obtained is denoted by $\vartheta(u_1, u_2)$ and later on by σ. The two first derivatives of ϑ with respect to u_i are the ζ-functions, and the three second derivatives with changed signs are the $\wp$-functions. All of these have properties strictly analogous to the corresponding functions in the elliptic case. The $\wp$-functions are expressible in a simple manner in terms of two positions on