SHORTER NOTICES.

La Geometria non-euclidea, esposizione storico-critica del suo Sviluppo. Da ROBERTO BONOLA. Bologna, Zanichelli, 1906. vi + 213 pp.

Wissenschaft und Hypothese IV : *Die nichteuklidische Geometrie, historisch-kritische Darstellung ihrer Entwicklung.* Von ROBERTO BONOLA, Professor an der Scuola Normale zu Pavia. Autorisierte deutsche Ausgabe besorgt von Prof. Dr. HEINRICH LIEBMANN. Leipzig, Teubner, 1908. viii + 244 pp.

IN the development of the sober science of mathematics a certain dramatic and even sensational element has been furnished by non-euclidean geometry, the history of which is therefore unusually interesting. By its very nature this subject lends itself easily to a historical and critical treatment, like that of the admirable book under review. Unfortunately, the very fascination of the subject has apparently retarded its growth along the substantial lines of actual detailed knowledge: the tendency has been to regard it as a curious and elegant plaything, rather than as the valuable adjunct to euclidean geometry, which it undoubtedly is. The slowness of its growth is illustrated by the fact that although more than eighty years have elapsed since Lobachevsky published his first epoch-making researches, it is only recently that quadric surfaces in non-euclidean space have been carefully studied and classified.

This and most other recent investigations are not mentioned by Bonola. Indeed, as he himself states, the character of his book is distinctly elementary. He begins by taking the reader back to the early period of questioning and doubt as to Euclid's fifth postulate, then carries him through the storm and stress period of creation by Gauss, Lobachevsky, and Bolyai, and is finally content to land him safely in the harbor of modern thought, where projective geometry, differential geometry, and the theory of continuous groups all afford cumulative evidence of the validity of the new doctrine.

In Chapter 5 the author gives several well-known methods of representing or imaging a non-euclidean space in a euclidean space, but omits to mention one introduced by Klein and oincaré and used successfully by Weber and Wellstein in their Encyklopädie der Elementar-Geometrie and by Liebmann (the