THE COLLINEATIONS OF SPACE.

Die Lehre von den geometrischen Verwandtschaften. Dritter Band: die eindeutigen linearen Verwandschaften zwischen Gebilden dritter Stufe. By RUDOLF STURM. Leipzig and Berlin, B. G. Teubner, 1909, viii + 574 pp.

THE third volume of Professor Sturm's treatise has about the same range of discussion for three dimensions as the preceding ones had for one and two respectively,* the size of the volume being explained by the very full treatment given to systems of correlations. This volume is not provided with a separate index nor a preface, but does contain a glossary of 165 technical words not found in the preceding ones, and the table of contents of all four volumes.

The first chapter (pages 1–352) is concerned with the ordinary problems of collineation and correlation in space, being somewhat similar to the treatment found in the corresponding parts of Reye's Geometrie der Lage, but much more extensive. The properties of central and axial perspective, affinity, similarity, and congruence are first brought out to discuss the collineations and polarity defined by linear complexes. All of this leads up to the theorem that a general collineation can be resolved into three axial involutions.

The two kinds of collineation which leave a given quadric invariant, and in particular a space cubic curve are given thirty pages. This could have been materially shortened by a more liberal use of analytic methods,† the development being almost exclusively synthetic. In fact the need of still another treatise, having the same general content as this first chapter, but proceeding algebraically, is keenly felt. It could be regarded as an appendix to the work of Professor Sturm.

Metrical, particularly focal, properties follow. Here a number of results are necessarily stated in algebraic form, but mostly without proofs. A knowledge of projective analytic geometry of quadrics is presupposed. Besides lines of curva-

*See the reviews in volume 15 of the BULLETIN ; that of Volume 1 on page 135, and of Volume 2 on page 252.

† As for example, that employed by Wiman ; "Ueber die algebraischen Curven von den Geschlechtern $p=4$, 5, and 6, welche eindeutige Transformationen in sich besitzen," *Bihang till Kongl. Svenska Vetenskaps-Akad. Handlingar*, 1895, and particularly, Miss Van Benschoten ; "Birational transformations of algebraic curves of genus four," *Amer. Jour. Math.*, vol. 31 (1909), pp. 213-252.