INFINITE SERIES.

Lehrbuch der unendlichen Reihen. Von DR. NIELS NIELSEN. Vorlesungen gehalten an der Universität Kopenhagen. Leipzig, Teubner, 1909. viii + 287 pp. Price 12 M.

THIS excellent elementary textbook shows the careful working over which is necessary to make a course of lectures fit the requirements of the classroom. From the beginning to the end the author constantly has in mind the students before him. The foundations do not presume a partial structure already in the students' minds, but start from the bare fundamentals all must possess to understand the course at all. For this reason the work is divided into three parts: theory of sequences, series with constant terms, series with variable terms. We may say that the first part defines what a series must mean, and what it can give us; the second part discusses the management of the particular value of a series for a given value of the variable; the third part discusses the sweep of values due to different values of the variable. The development proceeds leisurely and is well illustrated with examples. The references are sufficiently numerous to incite the student to follow up the subject in original papers, but not so exhaustive as to overwhelm him.

The first part contains six chapters, in order: rational numbers, irrational numbers, real sequences, complex sequences, applications to elementary transcendental functions, doubly infinite sequences. The conception of rational numbers and their combinations under the four processes of arithmetic is the beginning. From this is developed immediately the idea of rational sequence, and limit. It is then shown that any periodic decimal fraction represents a rational number, and that non-periodic decimals represent limits but not rational numbers. An irrational number a is defined to be the non-rational limit of two approximation series h_n, l_n, such that

$$h_n > a > l_n, \quad \lim (h_n - a) = 0, \quad \lim (a - l_n) = 0.$$

The irrational ω, $h_n > \omega > l_n$, is greater than the irrational ω', $h'_n > \omega' > l'_n$, if for a certain n, $l_n > h'_n$; and ω is less than ω', if for a certain n, $h_n < l'_n$. In any other case, for each n, $h_n \geqq l'_n$ and $h'_n \geqq l_n$ and $\omega = \omega'$. The four fundamental operations are defined for irrationals by defining the limits of $h_n \pm h'_n$, $l_n \pm l'_n$,