

4. One point of intersection, and contact of second order at a second point ;

5. Contact of third order.

Proper configurations exist for types 1 and 4, and do not exist for types 2, 3, and 5. For a given pencil of six conics there are four configurations for type 1, but only one for type 4.

For the one-dimensional case, we have the configuration $\Gamma_{5,1}^2$ on a line, and the pencil of quadrics is simply five pairs of points in involution.* We have two types, viz., the fixed points of the involution are distinct or they are coincident. Proper configurations exist for both cases. Given the five point pairs, there are two configurations determined in the first case, but only one in the second. This one-dimensional case is readily seen if the whole figure is projected upon a conic.

ON THE USE OF n -FOLD RIEMANN SPACES IN APPLIED MATHEMATICS.

BY PROFESSOR JAMES MCMAHON.

THE object of this article is to show that the conception of a Riemann surface has important physical bearings, and to indicate in a general way what kind of physical problems have been solved or may be solvable by the use of such n -fold surfaces or analogous manifold regions in three dimensions. The most recent work in this line constitutes the highest point yet reached in the application of modern function theory to physical problems. It is very noteworthy that a theory which was developed by following out purely intellectual relations, without any reference to the world of sense, should afterwards find unexpected applications and correspondences in the physical universe.

The conceptions of multiform functions, and of multiple spaces in which such functions are made uniform, furnish elegant solutions of some important problems in the theories of potential, electricity, light, sound, heat, and fluid motion. To give greater clearness to what follows, it may be well to take a simple illustration of a three-valued potential function in two dimensions, and show how to make it one-valued on a three-fold Riemann surface. Let (ρ, θ) be the polar coordinates of a

* Author's paper, loc. cit., p. 5, footnote.