$$T=x^6+y^6+z^6+3x^4y^2+3x^4z^2+3x^2y^4+3x^2z^4+\sum_{i=1}^{5} c_i y^{6-i}z^i+xyz\phi.$$

Set $y=\lambda z$. Then T becomes

$$t=x^6+r_2x^4z^2+r_3x^3z^3+\cdot+r_6z^6,$$

$$r_2=3\lambda^2+A\lambda+3,\ r_3=D\lambda^2+E\lambda,\ r_4=3\lambda^4+F\lambda^3+M\lambda^2+G\lambda+3,$$

$$r_5=B\lambda^4+K\lambda^3+L\lambda^2+C\lambda,\quad r_6=\lambda^6+c_1\lambda^5+\cdots+c_5\lambda+1.$$

Now $\tau=x^6\pm x^3z^3-z^6$, viz., is of type π, if and only if

$$(21)\qquad r_2\equiv r_4\equiv r_5\equiv 0,\quad r_3^2\equiv 1,\quad r_6\equiv -1\qquad (\text{mod } 7);$$

while τ is a perfect cube if and only if

$$(22)\qquad r_3\equiv r_5\equiv 0,\quad r_4\equiv 5r_2^2,\quad r_6\equiv 6r_2^3\qquad (\text{mod } 7).$$

Since $r_2r_3\equiv 0$ for every λ, $D\equiv E\equiv 0$. Hence (21) is excluded, so that (22) must hold for every λ. We may therefore remove the term y^5z from T and proceed as in § 10. Or we may proceed with (22) and show that

$$T\equiv(x^2+y^2+z^2-2Ayz)^3.$$

12. The theorem that there exists no sextic on three or more variables which represents only cubes in a field of order $p^n=3k+1$ has now been established for $p^n<31$, $p^n=2^n$, and $p^n=11^2$. Its truth for all values of p^n has been proved, subject to the validity of the conjectured theorem of § 9 on binary sextics.

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NOTE ON LÜROTH'S TYPE OF PLANE QUARTIC CURVES.

BY PROFESSOR H. S. WHITE AND MISS KATE G. MILLER.

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ONE of the stock examples of the fallacy of constant counting is the equation of a plane quartic, whose fourteen constants equal in number those apparent in the sum of five fourth powers of linear expressions

$$x_1^4+x_2^4+x_3^4+x_4^4+x_5^4.$$