

section elected Professor Winslow Upton, Ladd Observatory, as member of the sectional committee for five years.

G. A. MILLER,  
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SOME SURFACES HAVING A FAMILY OF  
HELICES AS ONE SET OF LINES OF  
CURVATURE.\*

BY MISS EVA M. SMITH.

IN a recent paper, Forsyth † gives a general method for the determination of surfaces with assigned lines of curvature, and he solves completely the case where both sets are circles. We apply the method to the case where one of the given sets consists of helices, and it appears that surfaces do exist having as one set of lines of curvature general helices ( $\rho/\tau = \text{constant}$  along each curve), but there are always limitations on the forms of  $\rho$  and  $\tau$ . In particular,  $\rho$  and  $\tau$  cannot both be constant along every curve. The complete solution seems to be too wide for analytic discussion, but there are two particular cases for which definite results can be obtained. This note contains a discussion of these cases.

1) Assuming that  $\rho$  and  $\tau$  are constant along each curve of the set (regular helices), we obtain the result that: *There are no surfaces with regular helices as one set of lines of curvature.*

2) If  $\rho/\tau$  is constant along each curve of one set of lines of curvature, and the other set consists of geodesics, we can obtain a complete solution; the equations of the resulting surfaces in parametric form are given at the end of this paper. The notation and equations used are those given in Darboux, *Théorie des surfaces*, volume 2, but derivatives with respect to  $u$  and  $v$  are here denoted by suffixes 1 and 2 respectively.‡

§ 1.

Consider the case where the helices are all regular. The curves  $v = \text{constant}$  are helices, and therefore  $\rho$  and  $\tau$  for these curves are functions of  $v$  only, and we denote  $\tau/\rho$  by  $k$ .

\* For the suggestion of this subject I am indebted to Prof. A. R. Forsyth.

† *Messenger of Mathematics*, vol. 38 (1908), pp. 33-44.

‡ Note that  $p_1$  is not  $\partial\rho/\partial u$ . To express derivatives of the rotations we use parentheses, e. g.  $(p)_1 = \partial p/\partial u$ .