

$$\int_a^b x^n \psi(x) dx = a_0 \int_a^b \psi(x) dx + a_1 \int_a^b \psi(x) \cos x dx \\ + b_1 \int_a^b \psi(x) \sin x dx + \dots = 0 \quad (n = 1, 2, \dots).$$

Hence  $\psi(x)$  satisfies the conditions of Theorem I, and therefore it is zero at every point at which it is continuous.

The method of proof used in Theorem II may be applied in the case of developments in terms of any other normal functions, such as Bessel functions, Legendre's polynomials, etc., whenever we know that  $x^n$  can be developed in a convergent series of such functions which, when differentiated term by term, will yield a uniformly convergent series that represents the derivative of  $x^n$ .\* The method will enable us to show in such cases that if the coefficients of the development corresponding to any function which we know to be finite, save for a finite number of points, and integrable, are all zero, the function is zero at every point at which it is continuous.

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## NOTE ON THE SECOND VARIATION IN AN ISOPERIMETRIC PROBLEM.

BY DR. ELIJAH SWIFT.

(Read before the American Mathematical Society, April 25, 1908.)

SUPPOSE we have before us the simplest type of isoperimetric problem, namely to determine  $x$  and  $y$  as functions of a parameter  $t$ , so that the definite integral

$$J = \int_{t_0}^{t_1} F(x, y, x', y') dt$$

shall be a minimum, while another definite integral

$$K = \int_{t_0}^{t_1} G(x, y, x', y') dt$$

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\* The existence of such developments can be proved for some of these cases by means of some theorems discussed by Stekloff. Cf. *Mémoires de l'Académie de St. Pétersbourg*, ser. 8, vol. 15 (1904).