ON CERTAIN CONSTANTS ANALOGOUS TO FOURIER'S CONSTANTS.

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IN the course of an article which appeared recently in the *Rendiconti del Circolo Matematico di Palermo*,* Landau has reproduced two proofs of the following theorem:

A. If the function $\psi(x)$ *is continuous in the interval* $(0 \leqq x \leqq 1)$ *and if*

$$\int_0^1 x^\nu \psi(x) dx = 0 \qquad (\nu = 0, 1, 2, \cdots), \tag{1}$$

then

$$\psi(x) = 0 \qquad (0 \leqq x \leqq 1).$$

The proofs that Landau gives in detail are due to Lerch and Stieltjes. In addition he cites a second proof due to Stieltjes and a proof due to Phragmen.

As far as I am able to learn, no one seems to have mentioned the fact that this theorem, of which so many proofs have been given, is essentially equivalent to a theorem due to Hurwitz† which may be stated as follows:

B. If in the interval $(0 \leqq x \leqq 2\pi)$ *the function* $f(x)$ *is finite and integrable and if all of its Fourier's constants are zero, then* $f(x)$ *is zero at every point of the interval at which it is continuous.*

Theorem (A) may be deduced from (B) as follows:

It is obvious that if $\psi(x)$ is finite and integrable in the interval $(0 \leqq x \leqq 1)$ and if condition (1) is fulfilled, then the function

$$f(y) = \psi(y/2\pi)$$

satisfies all the conditions of Hurwitz's theorem. For

* Vol. 25 (1908), p. 1.

† Cf. *Mathematische Annalen*, vol. 57 (1903), p. 440. Cf. also Bonnet, *Mémoires de l'Académie de Belgique*, vol. 23 (1850), p. 11.