$$(35) \quad \begin{aligned} 4\rho\sigma - \rho^2 - 3\sigma^2 - 4\sigma + 2\rho - 1 = 0, \\ (\rho - 1)(\rho - 1 - 3\sigma) = 0, \end{aligned}$$

the coefficient of λ^2 being zero. For $\rho = 1$, $\sigma = 0$, and the algebra is a field. For $\rho = 1 + 3\sigma$, (35_1) is satisfied; then $\kappa = -\rho$. Substituting (5′) in (4) and reducing by (3′), we find that the coefficients of λ^2 and λ vanish, and that the constant term is

$$-\sigma^2(\sigma + 1)(4d^3 + 27g^2) = 0.$$

But the second factor is not zero in view of the irreducibility of (3′). For $\sigma = 0$, the algebra is a field. For $\sigma = -1$, $\rho = -2$, and we obtain the non-field algebra

$$(36) \quad i^2 = j, \quad ij = ji = g - di, \quad j^2 = -d^2 - 8gi + 2dj.$$

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ISOTHERMAL SYSTEMS IN DYNAMICS.

BY PROFESSOR EDWARD KASNER.

(Read before the American Mathematical Society, October 26, 1907.)

CONSIDER any simply infinite system of plane curves defined by its differential equation

$$(1) \quad y' = f(x, y).$$

The ∞^2 isogonal trajectories satisfy the equation *

$$(2) \quad y'' = (F_x + y'F_y)(1 + y'^2),$$

where

$$F = \tan^{-1} f.$$

The theorem of Cesàro-Scheffers states that the trajectories passing through a given point have circles of curvature forming a pencil. We inquire whether any hyperosculating circles exist.

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* Primes are employed to denote derivatives with respect to x, and literal subscripts to denote partial derivatives.