THE INVARIANT SUBSTITUTIONS UNDER A SUBSTITUTION GROUP.

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WE begin with the case where the substitution group G is transitive and of degree n. If the subgroup G_1 composed of all the substitutions of G which omit a given letter is of degree $n-1$, there is no substitution which involves any of the letters contained in the substitutions of G and is also commutative with every substitution of G. In considering substitutions which are commutative with every substitution of G we shall confine ourselves to those which involve no letters that are not also contained in G, since every substitution which does not involve any of the letters of G is clearly commutative with every substitution of G. Hence we may say: *when the degree of G_1 is $n-1$, identity is the only substitution which is commutative with every substitution of G.*

When G_1 is identity it is well known that there are exactly n substitutions which are commutative with every substitution of G, and that these constitute a group which is simply isomorphic with G, known as the *associate* of G, whenever G is non-abelian.* It remains to consider the case where G_1 is of degree $n-\alpha$ $(1<\alpha<n)$. All the substitutions which transform G_1 into itself constitute a subgroup of order $\alpha g \div n$, g being the order of G. This subgroup may clearly be constructed by establishing a $(g_1, 1)$ isomorphism between a group of order $\alpha g \div n$ and a regular group of order α, g_1 being the order of G_1. If this regular group is abelian it is transformed into $n \div \alpha$ distinct regular groups by the substitutions of G and a simple isomorphism between these groups which is so constructed that all the conjugates of a given substitution correspond will be composed of all the substitutions which are commutative with every substitution of G. If the regular group of order α is non-abelian its associate is transformed in the manner stated. Hence we have the theorems: *The necessary and sufficient condition that there are substitutions besides identity which are com-*

* *Quar. Jour. of Math.* vol. 28 (1896), p. 249. When G is abelian it is self-associate.