

congruence of lines is reflected or refracted on any surface into a normal congruence. It is shown that each sphere of the congruence touches the focal surface in two points. The congruence of lines formed by joining these points is discussed and some very pretty relations between the surface of centers of the spheres and their envelope are derived therefrom. The chapter closes with a short account of the cyclical systems of Ribaucour and Weingarten surfaces.

An excellent set of exercises is given to accompany each chapter.
C. L. E. MOORE.

Nichteuklidische Geometrie. Von HEINRICH LIEBMAN. Leipzig, G. J. Göschen (Sammlung Schubert, XLIX), 1905. 12mo. viii + 248 pp.

IN this volume of the Schubert collection, Professor Liebmann has succeeded in presenting an introduction to non-euclidean geometry that is brief, readable, and well-balanced. Its brevity will recommend it to the student whose interest in the subject has been aroused by the numerous references in literature, but whose time and maturity are scarcely sufficient for a study of the many longer and more difficult works. It might well appeal also to a teacher of elementary geometry. The recent literature on non-euclidean geometry naturally falls into two classes: the one dealing with the lives and writings of Lobachevsky, Bolyai, and Gauss; and the other consisting of systematic developments of particular phases of the subject. In this book there is a happy combination of the two methods, giving a broad outlook, and yet not sacrificing the unity.

The first chapter contains an interesting account of the parallel axiom and of the attempts at its proof, considered from an historical point of view. The next five chapters, comprising three fourths of the book by pages, are devoted to hyperbolic geometry, beginning with a very simple account of its picturing by means of circles in the euclidean plane. One regrets that references are not given here to some, at least, of the articles that have appeared during the last twenty years on this picturing. With this one exception, the many exact references to a comparatively wide range of literature form one of the most attractive features of the book. There are other chapters on hyperbolic geometry, dealing with the synthetic and the analytic geometry and the trigonometry in the hyperbolic plane. After