

sider the cylinder as a special case of the cone, *i. e.*, a cone whose vertex is at infinity, for when  $a = \text{const.}$ ,  $b = \text{const.}$ , the conditions that the line generates a cone, viz.,  $p = ka + k_1$ ,  $q = kb + k_2$  become  $p = \text{const.}$ ,  $q = \text{const.}$  and the two consecutive generators actually coincide. Thus, we cannot say, that the shortest distance between two consecutive generators of a cylinder is zero, *i. e.*, that the two generators actually intersect. There is no shortest distance between two such lines; they are everywhere equally distant. Hence, to find the distance between two consecutive parallel lines, we shall have to use the formula for the distance of a point from a line. It is easily seen that, in general, this distance is an infinitesimal of the first order; it is zero only if the two consecutive lines coincide; it is infinite when  $a^2 + b^2 + 1 = 0$ . Hence

**THEOREM IX.** *The distance between two consecutive generators of a cylinder is, in general, an infinitesimal of the first order; if the generator is a minimal straight line, the distance is infinite.*

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## NOTE ON THE COMMUTATOR OF TWO OPERATORS.

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THERE is a confusing lack of uniformity with respect to the use of the term commutator. The present note aims to exhibit this fact and to point out some of its sources in the hope that these data may tend towards greater uniformity in the use of this term and also make its various meanings less confusing to the reader.

The operation now known as the commutator of two operators was used for a long time in the development of group theory before it received a special name. It is frequently employed, in various forms, in Jordan's *Traité des substitutions*, and its elegant application in the study of direct products was recognized by Hölder\* and others. The first paper which deals with the

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\*Hölder, *Math. Annalen*, vol. 34 (1889), p. 35. It should be noted that the reference 91) in *Encyklopädie der mathematischen Wissenschaften*, vol. 1, p. 219. should be to this article instead of to the later one in vol. 43.