SYSTEMS OF EXTREMALS IN THE CALCULUS OF VARIATIONS.

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THE extremals connected with any problem in the calculus of variations are the curves or surfaces or higher manifolds which satisfy the first of the conditions necessary for a maximum or minimum value. In the case of the integral of simplest type

$$(1) \qquad \int F(x, y, y')dx$$

this condition is given by the Euler-Lagrange equation

$$(2) \qquad \frac{d}{dx}F_{y'} - F_y = 0,$$

where the subscripts indicate partial differentiation. The expanded equation is

$$(3) \qquad F_{y'y'}y'' + F_{y'y}y' + F_{y'x} - F_y = 0.$$

The extremals here form a doubly infinite system of plane curves. Darboux's well-known investigation of the inverse problem shows that the system is entirely general *: Any doubly infinite system of curves may be regarded as the system of extremals belonging to some integral of type (1).

In this respect the simplest type of problem is unique. In all the more complicated integrals, involving either derivatives of higher order, or more than one dependent or independent variable, the Euler-Lagrange equations are of special form, and hence the system of extremals (curves or other manifolds) must possess peculiar geometric properties. Our object is to give an example, apparently the simplest, of such a property.

Consider the plane problem of second order connected with the integral

$$(4) \qquad \int F(x, y, y', y'')dx.$$

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* Darboux, Théorie des surfaces, vol. 3, no. 604.