los "* are here designated as "improper" continued fractions and their value determined by the following definition: The expression

$$(1) \qquad b_0 + \frac{a_1}{b_1} + \frac{a_2}{b_2} + \cdots + \frac{a_n}{b_n}$$

determines the quantities Z_k by the equations

$$(2) \qquad \begin{aligned} Z_n &= b_n, \quad Z_{n+1} = 1, \\ Z_m &= b_m Z_{m+1} + a_{m+1} Z_{m+2} \\ &\qquad (m = n-1, n-2, \cdots, 1, 0). \end{aligned}$$

The value of (1) is Z_0/Z_1. The only fractions which now fail to have values are those for which Z_1 is zero. If $Z_\kappa = 0$ for any value of κ other than 1, the fraction is improper. This definition leads to no new extension of the notion of an infinite continued fraction because improper convergents had been used previously even where the corresponding continued fractions were regarded as meaningless. Among the modern developments added to the chapter on infinite continued fractions, it is interesting to note the theorems of Van Vleck published in the second volume of the *Transactions*.

To sum up this notice and also the review of the first volume in last December's BULLETIN — the work as a whole gives an effect of conservatism, maturity and poise. It is not as likely as the modern French books to stimulate research, but it has a permanent value as a repository of accurate information about the conventional functions and processes of analysis.

OSWALD VEBLEN.

Lezioni sul Calcolo degli Infinitesimi, date nella R. Università di Modena da ETTORE BORTOLOTTI. Raccolte dal Dr. ARMANDO BARBIERI. Modena, 1905. vii + 61 pp.

BORTOLOTTI'S Calcolo degli infinitesimi is a short course of lectures on various questions that arise in connection with the determination of the relative orders of two infinitesimals (infinites)

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*Because $2 - \frac{1}{1} - \frac{1}{2} = 0$. The significance of this definition of "improper" continued fractions may perhaps be suggested by the equation

$$3 + \frac{2}{7} + \frac{5}{0} = 3 + \frac{2 \cdot 0}{0 \cdot 7 + 5} = 3.$$