dependent. With the aid of Theorem V it can be shown that a necessary and sufficient condition for the verification of the hypothesis of Bôcher's theorem is that $M_k(u_1, u_2, \cdots, u_n)$ be of constant rank $m < n$.

NORTHWESTERN UNIVERSITY,
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# SIGNIFICANCE OF THE TERM HYPERCOMPLEX NUMBER.

BY PROFESSOR JAMES BYRNIE SHAW.

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ESSENTIALLY four definitions of quite different logical import have been given for the term hypercomplex number, or multiple number. The four things so defined differ considerably in their mathematico-philosophical meaning, and while two of them are in a way equivalent, neither of the others can be correlated with these two or with each other as equivalent. It is proposed to examine these four definitions rather critically.

## I. *The n-tuple.*

The first definition I shall denominate the Dedekind definition, although Hamilton discussed, many years before, entities defined in the same way. It is of a pure arithmetical character, since it implies only the existence of a set of things we may call numbers, marks, or entities, according as we conceive them to belong to a domain of integrity, an abstract field, or, in general, an aggregate that we can call a range. At first these entities were in a scalar domain, then they were generalized to a rational domain, then to an abstract field, and obviously we may take them from any range. The definition runs substantially thus: *

A set of $n$ ordered marks (entities) $a_1, \cdots, a_n$ of a field (range) $F$, is called an $n$-tuple element $a$. The symbol $a = (a_1, \cdots, a_n)$ employed is purely positional without functional connotation. Its definition implies that $a = b$ if, and only if, $a_1 = b_1, \cdots, a_n = b_n$.

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* Dickson, "On hypercomplex number systems"; *Transactions Amer. Math. Society*, vol. 6 (1905), pp. 344-348.