NOTE ON THE HEINE-BOREL THEOREM.

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IN this note is presented a generalized form of the Heine-Borel theorem together with a corollary which is of immediate application in many theorems in metrical analysis. The present form of the theorem is the result of an effort to understand the meaning of the properties of boundedness and unboundedness of sets of points (numbers) in non-metrical analysis situs. It appears that for many purposes the property of boundedness when applied to a closed set may be replaced by the property that the set shall contain its limit points at infinity, *i. e.*, the set shall be closed even if it is unbounded. In non-metrical analysis, therefore, the chief distinction between a closed bounded set and an unbounded set not containing its limit points at infinity seems to be that the latter is necessarily not closed.

The theorem is stated for the case of three dimensions and the language of geometry is used exclusively. In view of the one-to-one correspondence of the set of all points in a three-space and the set of all triples of real numbers, the reader may, if he wishes, regard the geometric language as a notation for a three dimensional number manifold.

§ 1. *Definitions and Preliminary Notions.*

The word region is used to denote any set of points whatever. Two half-lines proceeding from the same point O and not lying in the same line form an angle. (The half-line contains the point O from which it proceeds.) We assume that an angle separates the remaining points of the plane in which it lies into two unique sets, an interior and an exterior set. If the half-lines a, b, c, no two of which lie in the same line and all three of which do not lie in the same plane, proceed from the same point O then the three angles formed by these half-lines together with the interior points of these angles form a trihedron $Oabc$. If A, B, C, are points of the respective half-lines a, b, c, then the four triangles OAB, OBC, OCA, and ABC, together with their interior points, form a tetrahedron. Such tetrahedron we shall speak of as associated with the tri-