$$N=\frac{1}{m_1}\left[\sum_{\sigma=0}^{(m_1-1)(p_2-2)} P(0,\,1,\,\cdots,\,p_2-2)^{m_1-1}\sigma+\psi\right],$$

where $P(0,\,1,\,\cdots,\,p_2-2)^{m_1-1}\sigma$ stands for the number of partitions of σ in (m_1-1)'s by the numbers $0, 1, \cdots, p_2-2$; and ψ is a determinate function of p_2 and m_1.

SPRINGFIELD, MO.,
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A DEFINITION OF QUATERNIONS BY INDEPENDENT POSTULATES.*

BY MISS R. L. CARSTENS.

(Read before the American Mathematical Society, February 24, 1906.)

§ 1. *Quaternions with respect to a Domain D.*†

THE usual theory relates to quaternions $a_1+a_2i+a_3j+a_4k$ in which the coefficients a_i range independently over all real numbers or else over all complex numbers, and the units have the following multiplication table:

	1	i	j	k
1	1	i	j	k
i	i	-1	k	$-j$
j	j	$-k$	-1	i
k	k	j	$-i$	-1

These conditions give the real quaternion system and the octonion system.‡ As an obvious generalization, the coefficients may range independently over all the elements of any domain D.

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*See Dickson, "On hypercomplex number systems," *Transactions Amer. Math. Society*, vol. 6 (1905).

† A domain consists of any class of elements.

‡ Octonions may be considered as quaternions with complex coefficients.